Logistic and exponential growth worksheet

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Paul Andersen explains how populations eventually reach a carrying capacity in logistic growth. He begins with a brief discussion of population size (N), growth rate (r) and exponential growth. He then explains how density dependent limiting factors eventually decrease the growth rate until a population reaches a carrying capacity (K). In this worksheet, we will practice using the logistic differential equation to model situations where the growth of a quantity is limited by a carrying capacity. Q1: A garden has a carrying capacity of 200 trees and needs a rate of 3% per month to be fully grown. If the initial number of fully grown trees in the garden is 20, what will the number of fully grown trees be after 9 months? Q2: An aquarium contains fish with a carrying capacity of 1,200 and a growth rate of 8%. If the initial population of the fish is 400, what is the population of the fish at any given time? AP(t)=4e1,200+e BP(t)=1,200e4+e CP(t)=2e1,200−e DP(t)=1,200e2+e EP(t)=1,200e2−e Q3: Bacteria are growing at a rate of 15% per minute in a closed container. If the initial number of bacteria is 2 and the carrying capacity of the container is 2 million cells, how long will it take the bacteria to reach 1 million cells? Give your answer to the nearest minute. Q4: The logistic population model assumes an upper limit L, beyond which growth cannot occur. The population y(t) has a rate of change that satisfies ddyt=ky1−yL for some positive constant k. A suitable function y(t) involves a second parameter b, determined by how rapid the initial growth is. Without integrating, which of the following could be y(t)? Ab1+Le BbLe−1 CL1+be DLbe−1 ELbe−1 Q5: Unlike exponential growth, where a population grows without bound, the logistic model assumes an upper limit L, beyond which growth cannot occur. The population y(t) has a rate of change that satisfies ddyt=ky1−yL for some positive constant k. Given a population with y(0)=L2, at what population is the growth zero? AL2 B0 Cnever DIt cannot be determined. EL Q6: The population of the citizens inside a village has a carrying capacity of 600 and a growth rate of 4%. If the initial population is 120 citizens, what is the population of the citizens in the village at any time? AP(t)=6e600+e BP(t)=600e6+e CP(t)=600e4−e DP(t)=4e600−e EP(t)=600e4+e Q7: A cage can hold up to 1,000 birds, and 200 birds are initially moved into the cage. Suppose that the population of birds grows according to the logistic model. If after 2 months there are 400 birds in the cage, after how many months will the population reach 800 birds? Give your answer to the nearest month. Q8: The biomass of Cerastium is assumed to follow a logistic growth model with an initial biomass of 0.1 g and a proportionality factor k=0.055, using day as a unit of time. At t=75days, a Cerastium plant's biomass was 3.0 g. Find the final biomass of this Cerastium plant when it is fully grown. Give your answer to one decimal place. Q9: The population growth of wolves in a national park follows a logistic growth model with an initial population of 15 wolves, a k-value of 0.05 (using a year as the unit of time), and a carrying capacity of 80. In roughly how many years does the model predict a wolf population of 60? Q10: It is assumed that the snail population in a kitchen garden can be modeled with a logistic growth model. At t=0, there were 6 snails in the patch. 1 month later, there were 12 snails. Given that the carrying capacity of the kitchen garden is 36 snails, at what time t in months does the model predict a snail population of 32? Round your answer to the nearest whole number of months. This lesson includes 7 additional questions and 72 additional question variations for subscribers. In this chapter, we have been looking at linear and exponential growth. Another very useful tool for modeling population growth is the natural growth model. This model uses base e, an irrational number, as the base of the exponent instead of \((1+r)\). You may remember learning about \(e\) in a previous class, as an exponential function and the base of the natural logarithm. The Natural Growth Model is \[P(t)=P_{0} e^{k t} onumber \] where \(P_{0}\) is the initial population, \(k\) is the growth rate per unit of time, and \(t\) is the number of time periods. Given \(P_{0} > 0\), if k > 0, this is an exponential growth model, if k < 0, this is an exponential decay model. a. Natural growth function \(P(t) = e^{t}\) b. Natural decay function \(P(t) = e^{-t}\) Figure \(\PageIndex{1}\): Natural Growth and Decay Graphs When a certain drug is administered to a patient, the number of milligrams remaining in the bloodstream after t hours is given by the model \[P(t) = 40e^{-.25t} onumber \] How many milligrams are in the blood after two hours? To solve this problem, we use the given equation with t = 2 \[\begin{align*} P(2) &= 40e^{-.25(2)} \\ P(2) &= 24.26 \end{align*} onumber \] There are approximately 24.6 milligrams of the drug in the patient's bloodstream after two hours. In the next example, we can see that the exponential growth model does not reflect an accurate picture of population growth for natural populations. Bob has an ant problem. On the first day of May, Bob discovers he has a small red ant hill in his back yard, with a population of about 100 ants. If conditions are just right red ant colonies have a growth rate of 240% per year during the first four years. If Bob does nothing, how many ants will he have next May? How many in five years? We solve this problem using the natural growth model. \[P(t) = 100e^{2.4t} onumber \] In one year, t = 1, we have \[P(1) = 100e^{2.4(1)} = 1102 \text{ ants} onumber \] In one year, t = 5, we have \[P(5) = 100e^{2.4(5)} = 16,275,479 \text{ ants} onumber \] That is a lot of ants! Bob will not let this happen in his back yard! Figure \(\PageIndex{2}\): Graph of Ant Population Growth in Bob's Yard. Note: The population of ants in Bob's back yard follows an exponential (or natural) growth model. The problem with exponential growth is that the population grows without bound and, at some point, the model will no longer predict what is actually happening since the amount of resources available is limited. Populations cannot continue to grow on a purely physical level, eventually death occurs and a limiting population is reached. Another growth model for living organisms in the logistic growth model. The logistic growth model has a maximum population called the carrying capacity. As the population grows, the number of individuals in the population grows to the carrying capacity and stays there. This is the maximum population the environment can sustain. \[P(t) = \dfrac{M}{1+ke^{-ct}} onumber \] where M, c, and k are positive constants and t is the number of time periods. Figure \(\PageIndex{1}\): Comparison of Exponential Growth and Logistic Growth The horizontal line K on this graph illustrates the carrying capacity. However, this book uses M to represent the carrying capacity rather than K. (Logistic Growth Image 1, n.d.) Figure \(\PageIndex{4}\): Logistic Growth Model (Logistic Growth Image 2, n.d.) The graph for logistic growth starts with a small population. When the population is small, the growth is fast because there is more elbow room in the environment. As the population approaches the carrying capacity, the growth slows. The population of an endangered bird species on an island grows according to the logistic growth model. \[P(t) = \dfrac{3640}{1+25e^{-0.04t}} onumber \] Identify the initial population. What will be the bird population in five years? What will be the population in 150 years? What will be the population in 500 years? We know the initial population,\(P_{0}\), occurs when \(t = 0\). \[P_{0} = P(0) = \dfrac{3640}{1+25e^{-0.04(0)}} = 140 onumber \] Calculate the population in five years, when \(t = 5\). \[P(5) = \dfrac{3640}{1+25e^{-0.04(5)}} = 169.6 onumber \] The island will be home to approximately 170 birds in five years Calculate the population in 150 years, when \(t = 150\). \[P(150) = \dfrac{3640}{1+25e^{-0.04(150)}} = 3427.6 onumber \] The island will be home to approximately 3428 birds in 150 years. Calculate the population in 500 years, when \(t = 500\). \[P(500) = \dfrac{3640}{1+25e^{-0.04(500)}} = 3640.0 onumber \] The island will be home to approximately 3640 birds in 500 years. This example shows that the population grows quickly between five years and 150 years, with an overall increase of over 3000 birds; but, slows dramatically between 150 years and 500 years (a longer span of time) with an increase of just over 200 birds. Figure \(\PageIndex{5}\): Bird Population over a 200-Year Span The student population at NAU can be modeled by the logistic growth model below, with initial population taken from the early 1960's. We will use 1960 as the initial population date. \[P(t) = \dfrac{30,000}{1+5e^{-0.06t}} onumber \] Determine the initial population and find the population of NAU in 2014. What will be NAU's population in 2050? From this model, what do you think is the carrying capacity of NAU? We solve this problem by substituting in different values of time. When \(t = 0\), we get the initial population \(P_{0}\). \[P_{0} = P(0) = \dfrac{30,000}{1+5e^{-0.06(0)}} = \dfrac{30,000}{6} = 5000 onumber \] The initial population of NAU in 1960 was 5000 students. In the year 2014, 54 years have elapsed so, \(t = 54\). \[P(54) = \dfrac{30,000}{1+5e^{-0.06(54)}} = \dfrac{30,000}{1+5e^{-3.24}} = \dfrac{30,000}{1.19582} = 25,087 onumber \] There are 25,087 NAU students in 2014. In 2050, 90 years have elapsed so, \(t = 90\). \[P(90) = \dfrac{30,000}{1+5e^{-0.06(90)}} = \dfrac{30,000}{1+5e^{-5.4}} = 29,337 onumber \] There are 29,337 NAU students in 2050. Finally, to predict the carrying capacity, look at the population 200 years from 1960, when \(t = 200\). \[P(200) = \dfrac{30,000}{1+5e^{-0.06(200)}} = \dfrac{30,000}{1+5e^{-12}} = \dfrac{30,000}{1.00003} = 29,999 onumber \] Thus, the carrying capacity of NAU is 30,000 students. It appears that the numerator of the logistic growth model, M, is the carrying capacity. Given the logistic growth model \(P(t) = \dfrac{M}{1+ke^{-ct}}\), the carrying capacity of the population is \(M\). \(M\), the carrying capacity, is the maximum population possible within a certain habitat. Suppose that in a certain fish hatchery, the fish population is modeled by the logistic growth model where \(t\) is measured in years. \[P(t) = \dfrac{12,000}{1+11e^{-0.2t}} onumber \] What is the carrying capacity of the fish hatchery? How long will it take for the population to reach 6000 fish? The carrying capacity of the fish hatchery is \(M = 12,000\) fish. Now, we need to find the number of years it takes for the hatchery to reach a population of 6000 fish. We must solve for \(t\) when \(P(t) = 6000\). \[6000 =\dfrac{12,000}{1+11e^{-0.2t}} onumber \] \[\begin{align*} (1+11e^{-0.2t}) \cdot 6000 &= \dfrac{12,000}{1+11e^{-0.2t}} \cdot (1+11e^{-0.2t}) \\ (1+11e^{-0.2t}) \cdot 6000 &= 12,000 \\ \dfrac{(1+11e^{-0.2t}) \cdot \cancel{6000}}{\cancel{6000}} &= \dfrac{12,000}{6000} \\ 1+11e^{-0.2t} &= 2 \\ 11e^{-0.2t} &= 1 \\ e^{-0.2t} &= \dfrac{1}{11} = 0.090909 \end{align*} onumber \] Take the natural logarithm (ln on the calculator) of both sides of the equation. \[\begin{align*} \text{ln} e^{-0.2t} &= \text{ln} 0.090909 \\ \text{ln}e^{-0.2t} &= -0.2t \text{ by the rules of logarithms.} \\ -0.2t &= \text{ln}0.090909 \\ t &= \dfrac{\text{ln}0.090909}{-0.2} \\ t&= 11.999\end{align*} onumber \] It will take approximately 12 years for the hatchery to reach 6000 fish. Figure \(\PageIndex{6}\): Fish Population over a 30-Year Period.

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